Answer the following questions and fill in your responses in the corresponding boxes on the answer sheet.

1. Fill in the blanks with the correct numbers.

   (1) If the equation $\sqrt{2}x^2 - \sqrt{3}x + k = 0$ with $k$ a constant has two solutions $\sin \theta$ and $\cos \theta \left(0 \leq \theta \leq \frac{\pi}{2}\right)$, then $k =$ \underline{ }.

   (2) Let $a$ be a real constant. If the constant term of $(x^3 + \frac{a}{x^2})^5$ is equal to $-270$, then $a =$ \underline{ }.

   (3) If the functions $f(x) = \frac{3x + 1}{2x + 1}$, $g(x) = \frac{px + 1}{2x - 3}$ satisfy the relation $f(g(x)) = x \left(x \neq -\frac{1}{2}, \frac{3}{2}\right)$, then the constant $p =$ \underline{ }.

   (4) The solution to the inequality $\log_2 x + \log_2(x - 2) < 4 \log_{16} 8$, in the set of real numbers, is \underline{ } < $x$ < \underline{ }.

   (5) The total number of positive divisors of 600 is \underline{ }, and the whole sum of those divisors is \underline{ }.
2. There are two circles, $C$ of radius 1 and $C_r$ of radius $r$, which intersect on a plain. At each of the two intersecting points on the circumferences of $C$ and $C_r$, the tangent to $C$ and that to $C_r$ form an angle of $120^\circ$ outside of $C$ and $C_r$. Fill in the blanks with the answers to the following questions.

(1) Express the distance $d$ between the centers of $C$ and $C_r$ in terms of $r$.

(2) Calculate the value of $r$ at which $d$ in (1) attains the minimum.

(3) In case (2), express the area of the intersection of $C$ and $C_r$ in terms of the constant $\pi$.

\[
\begin{array}{ccc}
(1) & (2) & (3) \\
\hline
\end{array}
\]

3. Consider the function $y = 8^x - 9 \cdot 4^x + 15 \cdot 2^x$ of $x$ ($-\infty < x < \infty$). Fill in the blanks with the answers to the following questions.

(1) Let $X$ denote $2^x$. Express $y$ in terms of $X$.

(2) Calculate the local maximum and minimum of $y$, and the values of $X$ in (1) at which $y$ attains them.

(3) Calculate the global maximum and minimum of $y$ in the interval $0 \leq x \leq \log_2 7$, and the values of $x$ at which $y$ attains them.

\[
\begin{array}{ccc}
(1) y = & \hline
(2) \text{The local maximum is } & \hline \\
\text{at } X = & \hline \\
\text{the local minimum is } & \hline \\
\text{at } X = & \hline \\
(3) \text{The global maximum is } & \hline \\
\text{at } x = & \hline \\
\text{the global minimum is } & \hline \\
\text{at } x = & \hline \\
\end{array}
\]