2009年度日本政府(文部科学省)奨学金留学生選考試験

QUALIFYING EXAMINATION FOR APPLICANTS FOR JAPANESE GOVERNMENT (MONBUKAGAKUSHO) SCHOLARSHIPS 2009

学科試験　　問題

EXAMINATION QUESTIONS

（高等専門学校留学生）

COLLEGE OF TECHNOLOGY STUDENTS

数　　学

MATHEMATICS

注意　☆試験時間は60分。

PLEASE NOTE : THE TEST PERIOD IS **60 MINUTES.**

MATHEMATICS

Nationality		No.	
Name	(Please print full name, underlining family name)		

Marks	

1 Fill in the blanks with correct numbers or expressions .

1) Solve the equation $16^x - 4^x - 2 = 0$.

2) Solve the equation $\sin x + 2\cos^2 x = 1,\quad (0 \leqq x < 2\pi)$.

3) Solve the inequality $x + \frac{1}{x} < \frac{1}{2}(7 - x)$.

4) Solve the inequality $\log_2(x+2) < 2$.

5) A number sequence $\{a_n\}$, $(n = 1, 2, 3, \cdots)$ satisfies the following conditions. Express a_n as a function of n .

$$3a_{n+1} = 2a_n + 1,\ (n = 1, 2, 3, \cdots),\qquad a_1 = 2 .$$

6) Let $f(x) = \cos x$ and $g(x) = \sin x$. Calculate

$$\lim_{h \to 0} \frac{f(x-2h) - f(x+h)}{g(x+3h) - g(x-h)}$$

7) Differentiate the function $e^{x \sin x}$.

8) Calculate $\int_{1/e}^{e} \log_e x \, dx$.

9) In a single toss of two dice, find the probability that the product of the two numbers is greater than their sum .

10) Find the real value of a such that the coefficient of x^9 is $\frac{21}{2}$ in the expansion of $\left(ax^2 - \frac{1}{ax}\right)^9$.

11) Let A and B be the points $(2, 0, 1)$ and $(0, 1, 2)$, respectively. Find the point P on the line through A and B such that $\overrightarrow{OP} \perp \overrightarrow{AB}$.

12) Let α and β be non-real roots of the equation $x^3 = 8$. Find the value of $\alpha^2 + \beta^2$.

2 Let $A=\begin{pmatrix}3 & 1\\ 2 & 4\end{pmatrix}$, $X=\begin{pmatrix}1\\ -1\end{pmatrix}$, $Y=\begin{pmatrix}1\\ 2\end{pmatrix}$ and $Z=\begin{pmatrix}3\\ 5\end{pmatrix}$.

1) Find the value of a which satisfies $AX=aX$.

2) Find the value of b which satisfies $AY=bY$.

3) Find the values of c and d which satisfy $Z=cX+dY$.

4) Calculate A^nZ.

3 Let k be a positive constant, $f(x) = |x^2 - k^2|$ and $I(k) = \int_{-1}^{1} f(x)\,dx$.

1) Sketch the graph of the function $y = f(x)$.

2) Suppose $k < 1$. Express $I(k)$ as a function of k .

3) Suppose $k > 1$. Express $I(k)$ as a function of k .

4) Find the minimum value of $I(k)$ and the value of k which minimizes $I(k)$.