

2009年度日本政府(文部科学省)奨学金留学生選考試験

QUALIFYING EXAMINATION FOR APPLICANTS FOR JAPANESE
GOVERNMENT (MONBUKAGAKUSHO) SCHOLARSHIPS 2009

学科試験 問題

EXAMINATION QUESTIONS

(学部留学生)

UNDERGRADUATE STUDENTS

数 学 (B)

MATHEMATICS (B)

注意 ☆試験時間は60分。

PLEASE NOTE : THE TEST PERIOD IS 60 MINUTES.

MATHEMATICS (B)

Nationality		No.		Marks	
Name	(Please print full name, underlining family name)				

1 Fill in the blanks with the correct numbers.

(1) Let ω be a solution of the equation $x^2 + x + 1 = 0$.

Then $\omega^{10} + \omega^5 + 3 = \boxed{}$.

(2) The constant term of $\left(2x^4 + \frac{1}{x^3}\right)^7$ is $\boxed{}$.

(3) The solution of the inequality $-x < x^2 < 2x + 1$ is $\{ \textcircled{1} < x < \textcircled{2} \}$.

(4) $\int_0^2 |x(x-1)| dx = \boxed{}$.

(5) If $\frac{1}{1-\sin\theta} + \frac{1}{1+\sin\theta} = 6$ and $0 < \theta < \frac{\pi}{2}$, then $\tan\theta = \boxed{}$.

2 Denote by D the domain

$$\{(x, y) \mid x \geq 0, y \geq 0\}.$$

Assume that a circle C contained in D touches the parabola $y = \frac{1}{2}x^2$ at the point $(2, 2)$ and also touches the x -axis. Find the radius of C .

3 Let A, B, C be three points on a plane and O be the origin point on this plane. Put $\vec{a} = \vec{OA}$, $\vec{b} = \vec{OB}$, and $\vec{c} = \vec{OC}$. P is a point inside the triangle ABC . Suppose that the ratio of the areas of $\triangle PAB$, $\triangle PBC$ and $\triangle PCA$ is $2 : 3 : 5$.

(1) The straight line BP intersects the side AC at the point Q .

Find $AQ : QC$.

(2) Express \vec{OP} in terms of \vec{a} , \vec{b} , \vec{c} .