

2014年度日本政府(文部科学省)奨学金留学生選考試験

QUALIFYING EXAMINATION FOR APPLICANTS FOR JAPANESE
GOVERNMENT (MONBUKAGAKUSHO) SCHOLARSHIPS 2012

学科試験 問題

EXAMINATION QUESTIONS

(高等専門学校留学生)

COLLEGE OF TECHNOLOGY STUDENTS

数 学

MATHEMATICS

注意 ☆試験時間は 60 分

PLEASE NOTE : THE TEST PERIOD IS 60 MINUTES.

MATHEMATICS

Nationality		No.		Marks	
Name	(Please print full name, underlining family name)				

1 Answer all questions and write your answers in the boxes provided.

1) If $x = \frac{\sqrt{5} + 2}{\sqrt{5} - 2}$ and $y = \frac{\sqrt{5} - 2}{\sqrt{5} + 2}$. Find the value of $x^2 + y^2$.

$$x^2 + y^2 =$$

2) Solve the equation $x^3 + 6x^2 + 11x + 6 = 0$.

$$x =$$

3) Solve the equation $\sin x - \cos x - \frac{1}{\sqrt{2}} = 0$ ($0 \leq x < 2\pi$).

$$x =$$

4) Solve the equation $2 \log_9(x + 2) + \log_3 x = 1$.

$$x =$$

5) Solve the inequality $2^{1-x} - 2^{x+2} < 7$.

6) Solve the inequality $\cos 2x + 9 \sin x - 5 < 0$ ($0 \leq x < 2\pi$).

7) Find the coordinates of the point A' which is the symmetric point of $A(3, 2)$ with respect to the line $2x + y - 12 = 0$.

8) Two coins are tossed at the same time. What is the probability that both coins show heads under the condition that at least one coin shows heads.

9) The sequence $\{a_n\}$ satisfies the following recurrence formula.

Express the general term a_n in terms of n .

$$a_1 = 1, \quad a_{n+1} = a_n + n + 1 \quad (n = 1, 2, 3, \dots)$$

$$a_n =$$

10) Find $\lim_{x \rightarrow 0} \frac{\tan x - \sin x}{x^3}$.

11) Find the derivative of the function $y = x \log_e \sqrt{x}$.

$$y' =$$

12) Find the indefinite integral $\int x e^{-2x} dx$.

2 Let $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, $O = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$. Let $A = \begin{pmatrix} 0 & -1 \\ 1 & a \end{pmatrix}$ satisfying $A^2 + A + I = O$.

Answer the following questions and write your answers in the boxes provided.

1) Evaluate a .

$a =$

2) Find A^3 .

$A^3 =$

3) Find $I + A + A^2 + \dots + A^{10}$.

$\left(\quad \quad \right)$

3 Let $I = \int_0^\pi (a \cos x + b \sin x + 1)^2 dx$, where a and b are real numbers.
Answer all questions and write your answers in the boxes provided.

1) Calculate $\int_0^\pi \cos^2 x dx$.

2) To calculate the integral in I , express I as a function a and b .

$I =$

3) Find the minimum value of I and the value of a and b which minimize I .

$\min I =$
 $a =$
 $b =$