MATHEMATICS (B) (2014)

Nationality		No.		Marks	
Name	(Please print full name, underlining family name)				

Answer the following questions and fill in your responses in the corresponding boxes on the answer sheet.

1. Fill in the blanks with the correct numbers.

(1) If the equation $\sqrt{2}x^2 - \sqrt{3}x + k = 0$ with k a constant has two solutions $\sin\theta$ and $\cos\theta$ $\left(0 \leq \theta \leq \dfrac{\pi}{2}\right)$, then $k =$ [ ].

(2) Let a be a real constant. If the constant term of $\left(x^3 + \dfrac{a}{x^2}\right)^5$ is equal to -270, then $a =$ [ ].

(3) If the functions $f(x) = \dfrac{3x+1}{2x+1}$, $g(x) = \dfrac{px+1}{2x-3}$ satisfy the relation $f(g(x)) = x$ $\left(x \neq -\dfrac{1}{2}, \dfrac{3}{2}\right)$, then the constant $p =$ [ ].

(4) The solution to the inequality $\log_2 x + \log_2(x-2) < 4\log_{16} 8$, in the set of real numbers, is [①] $< x <$ [②].

(5) The total number of positive divisors of 600 is [①], and the whole sum of those divisors is [②].

2. There are two circles, C of radius 1 and C_r of radius r, which intersect on a plain. At each of the two intersecting points on the circumferences of C and C_r, the tangent to C and that to C_r form an angle of $120°$ outside of C and C_r. Fill in the blanks with the answers to the following questions.

(1) Express the distance d between the centers of C and C_r in terms of r.

(2) Calculate the value of r at which d in (1) attains the minimum.

(3) In case (2), express the area of the intersection of C and C_r in terms of the constant π.

(1) [ ] (2) [ ] (3) [ ]

3. Consider the function $y = 8^x - 9 \cdot 4^x + 15 \cdot 2^x$ of x $(-\infty < x < \infty)$. Fill in the blanks with the answers to the following questions.

(1) Let X denote 2^x. Express y in terms of X.

(2) Calculate the local maximum and minimum of y, and the values of X in (1) at which y attains them.

(3) Calculate the global maximum and minimum of y in the interval $0 \leq x \leq \log_2 7$, and the values of x at which y attains them.

(1) $y =$ [ ].

(2) The local maximum is ① [ ] at $X =$ ② [ ];

the local minimum is ③ [ ] at $X =$ ④ [ ].

(3) The global maximum is ① [ ] at $x =$ ② [ ];

the global minimum is ③ [ ] at $x =$ ④ [ ].