

2009年度日本政府(文部科学省)奨学金留学生選考試験

QUALIFYING EXAMINATION FOR APPLICANTS FOR JAPANESE
GOVERNMENT (MONBUKAGAKUSHO) SCHOLARSHIPS 2009

学科試験 問題

EXAMINATION QUESTIONS

(学部留学生)

UNDERGRADUATE STUDENTS

数 学 (A)

MATHEMATICS (A)

注意 ☆試験時間は60分。

PLEASE NOTE : THE TEST PERIOD IS 60 MINUTES.

MATHEMATICS (A)

Nationality		No.	
Name	(Please print full name, underlining family name)		Marks

1 Fill in the blanks with the correct numbers.

- (1) Let α and β be solutions of $3x^2 - x - 3 = 0$.

Then $\alpha^2 + \beta^2 = \boxed{}$.

- (2) The solution of the inequality $-x < x^2 < 2x + 1$ is

$\boxed{\textcircled{1}} < x < \boxed{\textcircled{2}}$.

- (3) Let $\sin \alpha = \frac{1}{\sqrt{5}}$ ($0 < \alpha < 90^\circ$) and $\cos \beta = \frac{3}{\sqrt{10}}$ ($0 < \beta < 90^\circ$).

Then $\sin(\alpha + \beta) = \boxed{}$.

- (4) Let n be a natural number. If $3^n < 2^{100} < 3^{n+1}$, then $n = \boxed{}$.

Use $\log_3 2 = 0.631$.

- (5) The total number of pairs of integers (x, y) which satisfy the equation

$x^2 - 4xy + 5y^2 + 2y - 4 = 0$ is $\boxed{}$.

2 Let $f(a) = \int_0^2 |x(x-a)| dx$ for $0 \leq a \leq 2$.

- (1) Find the function $f(a)$.
- (2) Find the minimum of $f(a)$.

3 Let a be a real number such that $1 < a < 2$. $\{a_n\}$ is the sequence defined by

$$a_1 = a, \quad a_{n+1} = |a_n| - 1 \quad (n = 1, 2, 3 \dots).$$

And put $S_n = a_1 + a_2 + \dots + a_n$.

- (1) Find a_4, a_5, a_6, a_7 .
- (2) Find S_2, S_4, S_6 .
- (3) When $n = 2m$, where m is an integer ≥ 1 , express S_n in terms of a and m .
- (4) When $n = 2m + 1$, where m is an integer ≥ 1 , express S_n in terms of a and m .